

SCORE: ____ / 30 POINTS **3+**

1. NO CALCULATORS OR NOTES ALLOWED
2. SHOW PROPER PRECALCULUS-LEVEL WORK
3. SIMPLIFY ALL ANSWERS

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry.

SCORE: 3½ / 4 PTS

- [a] Write that identity involving $\sinh x$ and $\cosh x$. **NOTE: You do NOT need to prove the identity.**

$$\boxed{\cosh^2 x - \sinh^2 x = 1} \quad (1)$$

- [b] Divide both sides of that identity by $\cosh^2 x$ and simplify.

$$1 - \frac{\sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x} \Rightarrow \boxed{\operatorname{sech}^2 x = 1 - \tanh^2 x} \quad (1)$$

- [c] If $\sinh x = -7$, find $\coth x$.

$$\sinh x = -7 \quad \left(\frac{1}{2} \right) \quad \cosh^2 x = 1 + 7^2 = \sqrt{50} = 5\sqrt{2} \quad \left(\frac{1}{2} \right)$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{5\sqrt{2}}{-7} = \boxed{\frac{-5\sqrt{2}}{7}} \quad \left(\frac{1}{2} \right)$$

Write and **prove** a formula for $\sinh(x - y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$.

SCORE: 2 / 6 PTS

$$\sinh(x - y) = \sinh x \cosh y - \sinh y \cosh x \quad (2)$$

$$\begin{aligned} & \frac{e^x - e^{-x}}{2} - \frac{e^y - e^{-y}}{2} \\ & \downarrow \\ & \frac{e^x - e^{-x} - (e^y - e^{-y})}{2} \\ & \downarrow \\ & \frac{e^x - \frac{1}{e^x} - e^y + \frac{1}{e^y}}{2} \end{aligned}$$

Prove that $g(x) = \ln(x + \sqrt{x^2 - 1})$ is the inverse of $f(x) = \cosh x$ by simplifying $g(f(x))$.

SCORE: ____ / 5 PTS

$$\begin{aligned}
 g(f(x)) &= \ln(\cosh x + \sqrt{\cosh^2 x - 1}) \\
 &= \ln(\cosh x + \sqrt{\sinh^2 x}) \\
 &= \ln(\cosh x + \sinh x)
 \end{aligned}$$

$\cosh^2 x - \sinh^2 x = 1$
 $\cosh^2 x - 1 = \sinh^2 x$

Solve $\sinh x = 1$ by using the exponential definition of $\sinh x$ and an algebraic substitution $z = e^x$.

SCORE: 2.5 / 6 PTS

$$\sinh x = 1 = \frac{e^x - e^{-x}}{2} = 1$$

$$\begin{aligned}
 \sqrt{2} &\approx 1.4 \\
 1.4 &\times 2 \\
 \hline
 2.8
 \end{aligned}$$

$$\frac{z - \frac{1}{z}}{2} = 1$$

$$z - \frac{1}{z} = 2$$

$$z^2 - 1 = 2z$$

$$z^2 - 2z - 1 = 0$$

$$z = \frac{-(-2) \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 + 4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$\frac{2 + 2(1.4)}{2} = \frac{4.8}{2} = 2.4$$

$$\frac{2 - 2(1.4)}{2} = \frac{-0.8}{2} = -0.4$$

$$\boxed{\begin{matrix} z = -0.4 \\ z = 2.4 \end{matrix}}$$

Rewrite $\operatorname{csch}(3 \ln 2)$ in terms of exponential functions and simplify.

SCORE: ____ / 3 PTS

$$\operatorname{csch} = \frac{1}{\sinh} = \frac{2}{e^x - e^{-x}} = \frac{2}{e^{3 \ln 2} - e^{-3 \ln 2}} = \frac{2}{2^3 - 2^{-3}}$$

$$\begin{aligned}
 \frac{2}{8 - \frac{1}{8}} &= \frac{2}{\frac{64}{8} - \frac{1}{8}} = \frac{2}{\frac{63}{8}} = \frac{2}{1} \cdot \frac{8}{63} = \frac{16}{63}
 \end{aligned}$$

[MULTIPLE CHOICE] Write the letter of the correct answers in the spaces below.

ANSWERS:

[1]

d

[2]

d

[3]

f

[4]

d

[5]

e

[6]

a

[1] Which statement below regarding attendance is false ?

- ~~[a]~~ Whenever you come into class (whether on time or late), you should sign in on the attendance spreadsheet right away.
- ~~[b]~~ Arriving late on a quiz or midterm day will not be counted as late.
- ~~[c]~~ Unexcused early departures are considered absences.
- [d] If you have perfect attendance and classroom behavior for the first 7 weeks, and do not show up again after that, you will receive an F for the course.
- ~~[e]~~ Attendance policies will not apply to you if you score more than 80% on every midterm.

[2] Proper use of the textbook for this class includes

- [a] understanding all the terminology used in the book
- [b] working out the given examples yourself and checking that you are able to get the same results as the book
- [c] reading the sections of the textbook before the corresponding lecture
- [d] all of the previous answers [a], [b] and [c]
- [e] some, but not all, of the previous answers [a], [b] and [c]

[3] If you score 120 points on Midterm 1, 140 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value ?
(HINT: You are encouraged to start studying regularly early in the quarter.)

- [a] Midterm 1's score will be changed to 145 (the highest midterm score)
- [b] Midterm 1's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores)
- [c] Midterm 1's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 1's and Midterm 2's scores)
- [d] Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 1's and the highest midterm's scores)
- [e] Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ and Midterm 2's score will be changed to $(140 + 145) \div 2 = 142.5$ (the average of each midterm's and the highest midterm's score)
- [f] no midterm scores will be changed

[4] If you score 140 points on Midterm 1, 120 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value ?

- ~~[a]~~ Midterm 2's score will be changed to 145 (the highest midterm score)
- ~~[b]~~ Midterm 2's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores)
- ~~[c]~~ Midterm 2's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 2's and Midterm 1's scores)
- ~~[d]~~ Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 2's and the highest midterm's scores)
- ~~[e]~~ Midterm 1's score will be changed to $(140 + 145) \div 2 = 142.5$ and Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of each midterm's and the highest midterm's score)
- ~~[f]~~ no midterm scores will be changed

[5] Which statement below regarding tests (quizzes, midterms, final exam) is false ?

- ~~[a]~~ If you continue writing on your test after the stated ending time, you will receive a 0 for that test.
- ~~[b]~~ There are no make-ups for missed quizzes.
- [c] The instructor expects you to be able to identify and execute solutions on midterms more quickly than on quizzes because you should have had much more practice.
- [d] If your tablet, phone, computer etc. makes an audible noise during a test, you will lose 10% of all points available on that test.
- [e] If you cannot make the scheduled final exam time for any reason, your final exam can be rescheduled.

[6] How much of your learning does the instructor believe comes from your daily reading and homework combined ?

- [a] 40%
- [b] 50%
- [c] 60%
- [d] 70%
- [e] 80%